Journal of Engineering Physics and Thermophysics, Vol. 83, No. 6, 2010

MATHEMATICAL MODELING OF INSTABILITIES IN THE INTERACTION OF WAVE PROCESSES WITH THE CONTACT DISCONTINUITIES BETWEEN GASES OF DIFFERENT DENSITIES

A. V. Fedorov, V. M. Fomin, and G. A. Ruev

UDC 532.517.4:533.6.011.8

Works performed in the field of mathematical modeling of the process of mixing of gases of different densities on the interface between them under the action of transmitted and reflected shock waves and compression and rarefaction waves are reviewed. A mathematical model of two-velocity, two-temperature gases, which has been derived from the basic principles, is proposed for this modeling. A number of examples on the interaction of the above wave processes with the interfaces in helium–xenon and helium–argon mixtures are given; the appearing Richtmyer–Meshkov instability and the distinctive features of the wave dynamics of flow of a mixture are described. The performed comparison of the time dependence of the mixing-zone breadth and other dependences has shown that the description of the phenomenon within the framework of the proposed approach is satisfactory.

Keywords: *mixing of gases of different densities, laser thermonuclear fusion, contact discontinuity, shock waves, compression waves.*

Introduction. Problems of mathematical modeling of mixing, i.e., of the process appearing in interaction of the wave phenomena with the interfaces of two or more media of different densities, have attracted considerable attention from researchers [1–5]. This is due to the well-known technical applications of these processes, e.g., in problems of laser thermonuclear fusion, the theory of heterogeneous detonation where the initiating shock waves interact with the fronts of ignition and combustion of mixtures, leading to a mixing of the combustion products and the unignited mixture, etc. Interfaces between media of different densities are unstable to infinitesimal and finite perturbations, which gives mixing after the wave action. In certain cases this is an obstacle to realization of technical problems, whereas in others it contributes to their realization. Mathematical models for description of this phenomenon, except for the models of D. L. Youngs and V. F. Kuropatenko, are one-velocity models as a rule, i.e., are based on the assumption that both a heavy medium and a dense medium move with the same velocity and temperature. Mathematical models of the mechanics of homogeneous and heterogeneous two-velocity, two-temperature continua have been developed, over a period of years, at the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Russian Academy of Sciences as applied to the issues of the behavior of multicomponent solid bodies under moderate loads, to the problems of heterogeneous detonation, the theory of shock waves in binary and ternary gaseous mixtures, etc. A mathematical model of gases of different densities, dating back to V. V. Struminskii and derived from the basic principles, was used for this purpose. Below, we develop this model as applied to certain problems of unstable behavior of a contact discontinuity.

Thus, we consider, in 1*D* and 2*D* approximations, the evolution of a mixing layer of two gases of different densities on exposure to shock waves and compression waves. Passage of the shock wave from one gas into the other through a perturbed contact discontinuity generates a Richtmyer–Meshkov instability. In the final stage, a turbulent mixing zone separating compressed-gas flows is formed in the region of the initial contact discontinuity. We will describe this process on the basis of multivelocity, multitemperature models [6–14].

Evolution of the Diffusion Layer of Mixing of Two Gases in Its Interaction with Shock Waves in One-Dimensional Nonstationary Flow of the Mixture. *Basic system of equations.* The parameters of the mixture in the

---

Khristianovich Institute of Theoretical and Applied Mechanics, Siberian Branch of the Russian Academy of Sciences, 4/1 Institutskaya Str., Novosibirsk, 630090, Russia; email: fedorov@itam.nsc.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 6, pp. 1142–1148, November–December, 2010. Original article submitted April 28, 2010.

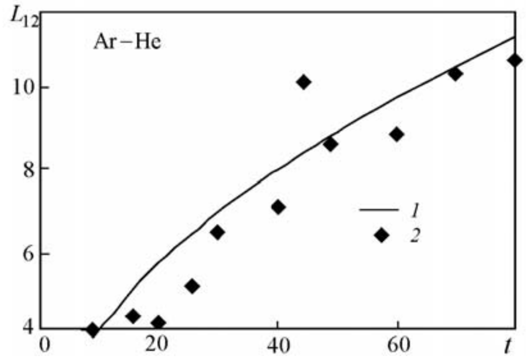


Fig. 1. Growth in the diffusion breadth of the mixing zone with time: 1) calculation; 2) experimental data [15]. L_{12}, mm; t, μs.

mixing layer of two gases of different densities are described by the equations of two-velocity, two-temperature gasdynamics of mixtures [6]:

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_i}{\partial x}=0\,,\quad \rho_i\frac{\partial u_i}{\partial t}+\rho_i u_i\frac{\partial u_i}{\partial x}+\frac{\partial p_i}{\partial x}=K\,(u_j-u_i)\,,$$

$$\rho_i\frac{\partial e_i}{\partial t}+\rho_i u_i\frac{\partial e_i}{\partial x}+p_i\frac{\partial u_i}{\partial x}=K\beta_{ij}\,(u_j-u_i)^2+q\,(T_j-T_i)\,,\tag{1}$$

$$p_i=kn_iT_i\,,\quad e_i=\frac{k}{m_i\,(\gamma_i-1)}\,T_i\,,\quad \rho_i=m_in_i\,,\quad i,j=1,2\,,\quad i\neq j\,.$$

Here $K=\dfrac{16}{3}\dfrac{\rho_1\rho_2}{m_1+m_2}\Omega_{12}^{(1,1)}$, $\Omega_{12}^{(1,1)}$ is the collision integral; $\beta_{ij}=\dfrac{m_iT_i}{m_iT_i+m_jT_j}$, $q=\dfrac{3m_1K}{m_1+m_2}$, $c_{iv}=\dfrac{k}{m_i(\gamma_i-1)}$, and γ_i is the adiabatic exponent. For the interaction potential of solid spheres, K is represented in the form

$$K=\frac{16}{3}\frac{\rho_1\rho_2}{m_1m_2}\sqrt{k\pi/2}\sqrt{\frac{T_1}{m_1}+\frac{T_2}{m_2}}\,\delta_{12}^2\,,\quad \delta_{12}=(\delta_1+\delta_2)/2\,.$$

Formation of the mixing zone. In experimental works (see references in [1–5, 15]), a fast-removable plate dividing the channel into two parts was used for creation of the initial mixing zone in a shock tube. The basic mechanism responsible for mixing is molecular diffusion. For description of the initial mixing zone, an asymptotic solution was obtained from (1) for $K >> 1$:

$$u=0\,,\quad T=\text{const}\,,\quad n=\text{const}\,;$$

$$x_1=(1-\Phi\,(\eta))/2\,,\quad \Phi\,(\eta)=\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-u^2}\,du\,,\quad \eta=\frac{x}{2\sqrt{Dt}}\,,\tag{2}$$

this solution satisfies the initial distribution of the molar concentration: $x_1 = 1$ at $x < 0$ and $x_1 = 0$ at $x > 0$.

Following [1], we introduce the mixing-zone breadth in terms of molar concentration in the form

$$L=2\int_{-\infty}^{x_0(t)}\frac{x_2-x_2^0}{x_2\,(x_0\,(t))-x_2^0}\,dx+2\int_{x_0(t)}^{+\infty}\frac{x_1-x_1^1}{x_1\,(x_0\,(t))-x_1^1}\,dx\,,\tag{3}$$

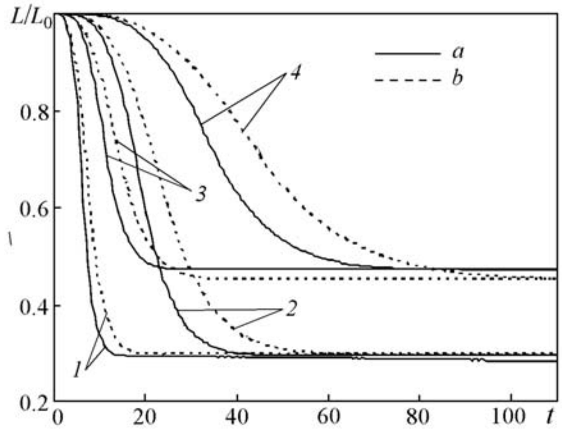


Fig. 2. Distribution of the relative breadth of the mixing zone with time: a) He–Ar; b) He–Xe; 1) M = 2.5 and L_0 = 13, 2) 2.5 and 40, 3) 1.5 and 13), and 4) 1.5 and 40 mm. t, μs.

where x_i^0 and x_i^1 are the molar concentrations of the ith gas on both sides of the mixing zone and $x_0(t)$ is the center of the mixing zone, i.e., the point at which we have $x_1 = x_2 = (x_i^0 + x_i^1)/2$. For the solution (2), we obtain

$$L(t) = 4\left[\int_{-\infty}^{0}(1 - x_1)\,dx + \int_{0}^{+\infty} x_1 dx\right].$$

Figure 1 gives a comparison to experimental data [15] on the growth in the diffusion breadth of the mixing layer in the Ar–He mixture. It is seen that formulas (2) and (3) satisfactorily describe the experimental data. Calculations from the complete model coincide with the asymptotic solution (2), which enables us to begin to calculate the interaction with the shock wave, taking this asymptotic solution as the initial parametric distribution in the layer.

Interaction of the Diffusion Layer with the Shock Wave. We consider the transmission of the shock wave by a layer generated by molecular diffusion. At the initial instant of time, there is a mixing layer of two gases in the zone $-L_0/2 < x < L_0/2$ (gas 1 on the left and gas 2 on the right). A shock wave with Mach number M, which is at the point $x = L_0/2$ at the instant $t = 0$, is incident on the layer on the right.

Passage of the shock wave from a light gas into a heavy gas. Passage of the shock wave from He into Ar (with Atwood number $A = \frac{\rho_h - \rho_l}{\rho_h + \rho_l} = 0.82$, where ρ_h is the density of the heavy gas and ρ_l is the density of the light gas at the initial instant of time) and from He into Xe is investigated. Figure 2 gives the changes in the relative diffusion breadth of the mixing zone L/L_0 as functions of the time. It is seen that the total compression of the layer is mainly determined by the Mach number and is weakly dependent on the ratio of the molecular weights and the initial breadth of the layer. As the Mach number changes from 1.5 to 2.5, the layer thickness diminishes 2.2 to 3 times. Increase in the initial breadth of the layer influences the compression time, which is due to the longer period of transmission of the shock wave by the layer. A decrease of 2–2.5 times in the layer thickness is noted in the experiments of [15], too. Consideration is also given to the passage of the shock wave from the heavy gas Xe into the light He. It turns out that for large Atwood numbers, the layer becomes strongly pressed after the passage of the shock wave and expands in such a manner that it ends up being compressed approximately 1.5 times. The effect of expansion after the compression disappears on decrease in the molecular-weight ratio. As in the case where the shock wave passed from the light gas into the heavy one, the initial breadth of the layer influences only the compression time. We have an analogous behavior of the mixing-zone thickness for other Mach numbers. Investigation of the interaction of the layer with the compression wave shows that the calculation data agree with the experimental data on the change in the relative breadth of the diffusion-mixing zone for passage of the compression wave from the mixture into Ar (from the light gas into the heavy one). In the calculations, unlike the experiment, we have slower compression of the gas behind the transmitted shock wave and stronger compression behind the shock wave reflected from the end.

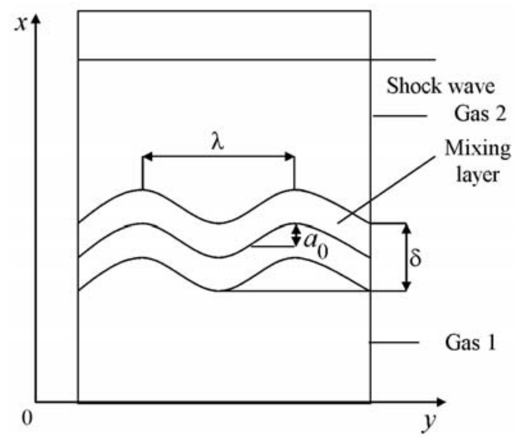


Fig. 3. Physical pattern of flow in interaction of the shock wave with the contact discontinuity.

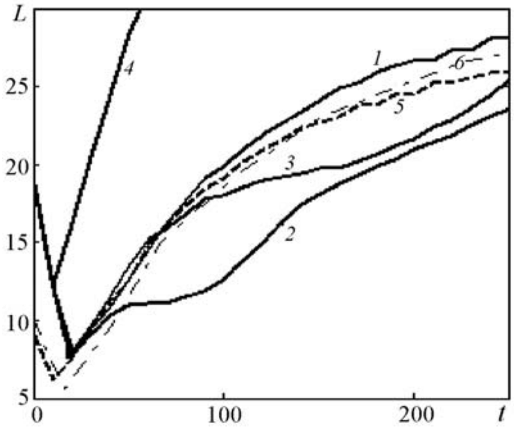
Interaction of the Diffusion Layer with the Shock Wave in a Two-Dimensional Channel. Let us investigate the evolution of a transition layer separating two pure gases with different densities within the framework of the model of two-dimensional nonstationary flow of a two-velocity, two-temperature mixture when the layer is acted upon by a shock wave. Formulation of the problem and the notation are given in Fig. 3. The well-known method of splitting of the flux vector is used as the calculation method for spatial approximation of system (1).

Passage of the shock wave from a light gas into a heavy one. It is common knowledge that when a shock wave passes from a light gas into a heavy one through a perturbed contact discontinuity refracted and reflected shock waves are observed. The forms of these waves are similar to the form of the perturbation of the initial interface. An analogous situation is observed in passage of the shock wave through a perturbed mixing layer. However, the presence of the transition zone leads to a delay in the perturbation growth and to certain features of the development of a Richtmyer–Meshkov instability.

Analysis of the isolines of total pressure of the mixture at different instants of time in passage of the shock wave from argon into xenon has shown that when the shock wave passes from the light gas into a heavy one the form of the refracted shock wave is similar to the form of the initial perturbation of the mixing layer. The interaction of the shock wave with the mixing layer produces a heavy-gas jet directed toward the light gas. From a certain instant of time, a mushroom-shaped structure begins to form on the lateral boundaries of the jet, i.e., a Kelvin–Helmholtz instability develops. In due time, we have rectification of the refracted and reflected waves whose parameters tend to the values obtained above for the one-dimensional case.

It turns out that as the perturbation wavelength diminishes, we have earlier formation of the mushroom-shaped structure on the jet surface and more intense expansion of the jet in the transverse direction, which leads to an interaction of the neighboring jets. Until vortices appear, the mixing layer is rather thin and can be considered as the discontinuity surface. However, the development of the vortex structure leads to an intense mixing and increase in the breadth of the layer, which makes it impossible to consider it as the discontinuity surface. The depth of penetration of the heavy gas into the light one grows with molecular-weight ratio.

Figure 4 shows the change in the total breadth of the mixing layer with time for different perturbation wavelengths and Atwood numbers. The shock wave passes from the light gas (argon or helium) into xenon. After the compression of the mixing layer by the shock wave, a smaller perturbation wavelength (curves 1 and 3) corresponds to a faster increase in the layer breadth at the linear stage of development of a Richtmyer–Meshkov instability. But at the nonlinear stage, the decrease in λ leads to a reduction in the rate of growth in the mixing-layer thickness. This is due to the more intense expansion of the layer in the transverse direction with decrease in the wavelength. The case of small λ values (curve 2) is characterized by the absence of the linear stage, since the formation of the heavy-gas jet and the development of vortices begin when the shock wave is still in the layer. As the molecular-weight ratio increases, the time of the beginning of growth in the perturbation amplitude diminishes and the rate of growth in the layer breadth increases (curve 4). Also, Fig. 4 gives results of calculation (curve 5) for the small initial diffusion



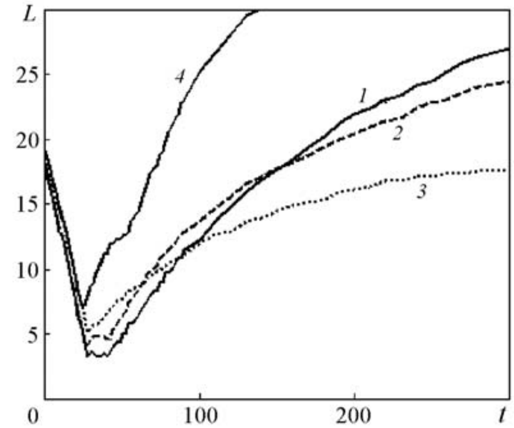


Fig. 4. Change in the total breadth of the mixing layer with time for different values of λ and δ_0 at M = 3.5 and a_0 = 5 mm: 1–3) Ar $\to$ He, δ_0 = 10 mm [1) λ = 36, 2) 12, and 3) 24 mm]; 4) He $\to$ Ar, λ = 36 mm and δ_0 = 10 mm; 5) Ar $\to$ He, λ = 36 mm and δ_0 = 1 mm; 6) calculation results from [19]. L, mm; t, μs.

Fig. 5. Change in the total breadth of the mixing layer with time for different values of λ at M = 3.5: 1–3) Xe $\to$ Ar [1) λ = 36, 2) 24, and 3) 12 mm; 4) Xe $\to$ He, λ = 36 mm. L, mm, t, μs.

breadth of the layer (δ_0 = 1 mm); these results are similar to the results of calculation based on Euler equations (curve 6), which have been obtained in [16–18] for the density profile discontinuous at the initial instant.

Passage of the shock wave from a heavy gas into a light one. Figure 5 plots L versus t for different values of the wavelength. At the initial stage of development of the instability, a smaller perturbation wavelength corresponds to a faster increase in the layer breadth. With time, the decrease in the perturbation wavelength leads to an earlier development of vortex structures and to an intense growth in the mixing layer in the transverse direction and a decrease in the layer breadth in the longitudinal direction (curves 1 and 3). At M = 3.5, the increase in the molecular-weight ratio (curve 4, λ = 36 mm) leads to a deeper penetration of the jet of the heavy gas (Xe) into the light one (He).

Development of the Richtmyer–Meshkov Instability in Interaction of the Diffusion Layer of Mixing of Two Gases with the Transmitted and Reflected Shock Waves. Let us investigate the process of development of a Richtmyer–Meshkov instability in interaction of the incident shock wave with the perturbed mixing layer near the end of a shock tube, i.e., with allowance for further multiple interaction of the layer with the waves reflected from the end. In this case, the shock wave moves from the top in a downward direction. The condition of equality of the derivatives to zero is set on the upper boundary; the lower boundary is a solid wall. The symmetry conditions are set on the lateral boundaries. At first, we consider the incidence of the shock wave on the mixing layer with allowance for reflection in a one-dimensional approximation in the absence of the perturbations of the initial mixing layer. The results of these investigations are in complete agreement with the conclusions of [19]. Further investigation is carried out in a two-dimensional approximation.

Consideration is given to the passage of the shock wave from air (light gas) into SF_6 (heavy gas) with a Mach number of 1.32; the distance from the center of the mixing layer to the end is 10 cm at the initial instant of time, and the initial breadth of the layer is 15 mm. Figure 6 gives the isolines of molar concentration of the heavy gas SF_6 at different instants of time after the beginning of the interaction with the incident shock wave.

As a result of the interaction of the shock wave with the mixing layer, it contracts, and once the shock wave has emerged from the layer, the perturbation amplitude begins to grow, as has been described earlier in [20]. Thereafter the layer interacts with the shock wave reflected from the end, which leads to a rectification of the layer followed by a change of the perturbation phase (the reflected wave moves from the heavy gas to the light one now) and to a sharper growth in the layer breadth. Next, there appears a jet of heavy gas and a mushroom-shaped structure is subsequently formed.

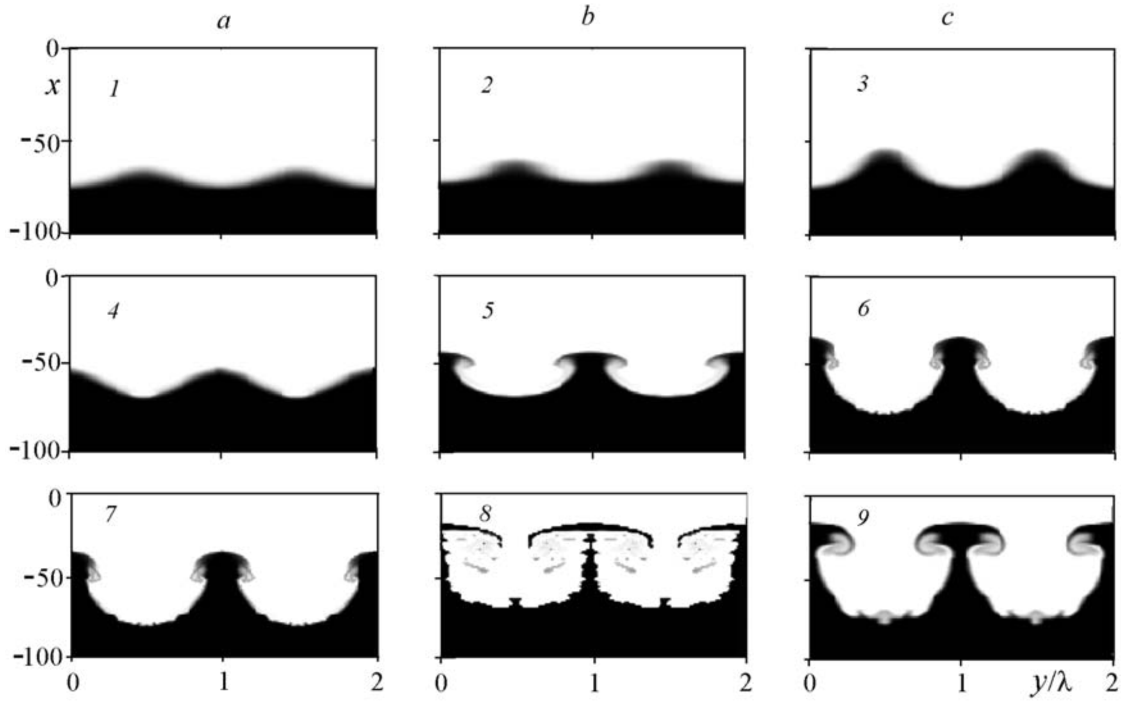


Fig. 6. Isolines of molar concentration of SF_6: a) a_0 = 1 and λ = 60, b) 1 and 30, and c) 3 mm and 60 mm; 1–3) t = 0.7, 4–6) 1.2, and 7–9) 2 μs. x, mm.

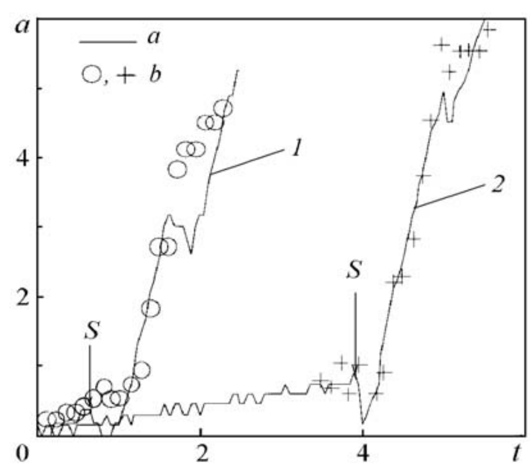


Fig. 7. Perturbation amplitude vs. time for different distances from the mixing layer to the end: a) calculated data; b) experiment [20]; 1) the distance from the center of the mixing layer to the end at the initial instant of time is equal to 10 cm, a_0 = 0.1 mm, λ = 6 cm, and δ_0 = 15 mm; 2) 55 cm, 0.05 mm, 6 cm, and 15 mm. t, μs.

Figure 7 shows a comparison to the experimental data of [20] on the rate of growth in the perturbation amplitude. The shock wave propagates from air into SF_6 (M = 1.32). It is seen that the perturbation amplitude sharply grows after the interaction of the reflected shock wave with the mixing layer.

Consideration is also given to the passage of the incident shock wave from a heavy gas to a light one. In this case the incident shock wave passes from the heavy gas into the light gas and the perturbation phase is changed before the arrival of the reflected shock wave. Thereafter, a jet of heavy gas is formed by the additional action of the reflected compression waves to form a mushroom-shaped structure. In the case of the passage of the shock wave from

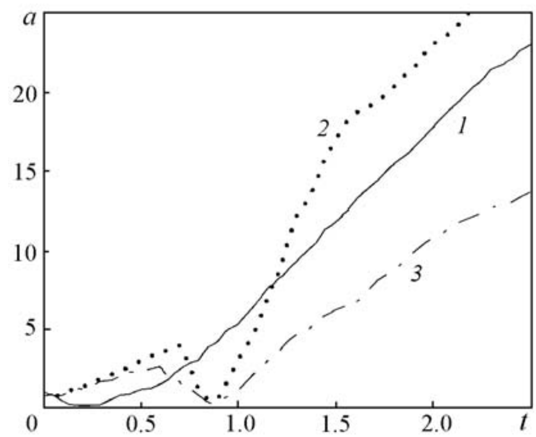


Fig. 8. Perturbation amplitude vs. time for an initial distance to the wall of 10 cm, a_0 = 1 mm, λ = 6 cm, δ_0 = 15 mm, and M = 1.32 for the shock wave; 1) from SF_6 into air; 2) from air into SF_6; 3) from argon into xenon.

the light gas into the heavy one we have a perturbation growth more intense than that in the case of the passage of the shock wave from the heavy gas into the light gas (curves 1 and 2 in Fig. 8).

Figure 8 gives results of calculation of the growth in the perturbation amplitude in passage of the shock wave from argon into xenon (molecular-weight ratio 5.03, curve 3, whereas for air and SF_6, the molecular-weight ratio is 3.28, curve 2). It is seen that the growth in the perturbation amplitude increases with molecular-weight ratio.

Conclusions. The mathematical model of a two-velocity, two-temperature mixture of gases with highly differing molecular weights has been applied to the description of processes occurring in the interaction of shock waves with the mixing zone of two gases which is perturbed in a sine manner. The performed analysis of the appearing wave patterns of flow in passage of the shock wave both from a light gas into a heavy one and from a heavy gas into a light one has shown that allowance for the initial finite breadth of the mixing layer leads to a decrease in the rate of growth in the relative layer breadth. It has been found that on the jet boundaries, there appear vortices with the center in the mixing layer, where each component is characterized by its own velocity. The appearance of such vortex formations leads to an intense expansion of the layer in the transverse direction, which in turn causes the neighboring jets to interact with each other. The mathematical model has been verified from the results of measurements of the growth in the perturbation amplitude of the mixing layer.

We have described processes occurring in interaction of the shock waves with the mixing zone of two gases which is perturbed in a sine manner with allowance for the multiple reflection of the waves from the end. We have analyzed the appearing wave patterns of flow where either the light gas or the heavy gas is near the wall; the flow patterns turned out to be totally different. The performed comparison of the calculation data and the results of measurement of the growth in the perturbation amplitude of the mixing layer has shown that the mathematical model is also adequate for phenomena occurring in semiinfinite zones.

NOTATION

a_0, perturbation amplitude; c_v, specific heat at constant volume; D, wave-front velocity; e_i, internal energy of the ith molecule; k, Boltzmann constant; L, diffusion breadth of the mixing zone; M, Mach number; m_i, weight of the ith molecule; n, molecular number; p_i, pressure of the ith molecule; S, time of first contact of the reflected shock wave with the layer; T_i, temperature of the ith molecule; t, time; u_i, velocity of the ith molecule; x, space coordinate; δ_i, diameter of the ith-gas molecule; δ_0, initial diffusion breadth of the layer; λ, perturbation wavelength; ρ_i, density of the ith molecule.

REFERENCES

1. V. E. Neuvazhaev, *Mathematical Simulation of Turbulent Mixing* [in Russian], Izd. RFYaTs–VNIITF, Snezhinsk (2007).
2. N. A. Inogamov, A. Yu. Dem'yanov, and É. E. Son, *Hydrodynamics of Mixing* [in Russian], Izd. MFTI, Moscow (1999).
3. O. Belotserkovskii, *Turbulence and Instabilities* [in Russian], MZ-Press, Moscow (2003).
4. E. E. Meshkov, *Investigation of Hydrodynamic Instabilities under Laboratory Conditions* [in Russian], FGUP "RFYaTs–VNIIÉF," Sarov (2006).
5. D. L. Youngs, Numerical simulation of mixing by Rayleigh–Taylor and Richtmyer–Meshkov instabilities, *Laser and Particle Beams*, **12**, No. 4, 725–750 (1994).
6. S. P. Kiselev, G. A. Ruev, A. P. Trunev, V. M. Fomin, and M. Sh. Shavaliev, *Shock-Wave Process in Two-Component and Two-Phase Media* [in Russian], Nauka, Novosibirsk (1992).
7. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Evolution of the diffusion layer of mixing of two gases in its interaction with shock waves, *Prikl. Mekh. Tekh. Fiz.*, **45**, No. 3, 24–31 (2004).
8. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Interaction of shock waves with mixing layer of two gases with widely differing masses of molecules, in: *Proc. 12th Int. Conf. Methods of Aerophysical Research* (ICMAR 2004), Vol. 1, ITAM, Novosibirsk (2004), pp. 194–197.
9. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Development of the Richtmyer–Meshkov instability due to the interaction of a diffusion mixing layer with shock waves, *Prikl. Mekh. Tekh. Fiz.*, **46**, No. 3, 3–11 (2005).
10. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Development of the Richtmyer–Meshkov instability due to the interaction of a diffusion layer of mixing of two gases with transmitted and reflected shock waves, *Dokl. Ross. Akad. Nauk*, **427**, No. 4, 489–491 (2009).
11. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Development of the Rayleigh–Taylor instability due to interaction of a diffusion mixing layer of two gases with compression waves, *Shock Waves*, **16**, No. 1, 65–74 (2006).
12. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Evolution of diffusion mixing layer of two gases at its interaction with compression waves, in: *AIP Conference Proceedings*, **849**, 311–316 (2006).
13. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Modeling of the development of Rayleigh–Taylor and Richtmyer–Meshkov instabilities on the basis of the multivelocity multitemperature gasdynamics of mixtures, in: *IX Zababakhin Scientific Lectures*, *Abstracts of papers submitted to the Int. Conf. on the Physics of High Energy Densities*, 10–14 September 2007, RFYaTs–VNIITF, Snezhinsk (2007), pp. 149–150.
14. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Description of the anomalous Rayleigh–Taylor instability on the basis of the model of the three-velocity and three-temperature dynamics of mixtures, *Prikl. Mekh. Tekh. Fiz.*, **50**, No. 1, 58–67 (2009).
15. S. G. Zaitsev, S. N. Titov, and E. I. Chebotarev, Evolution of the transition layer separating different-density gases with the shock wave transmitted by it, *Izv. Akad. Nauk SSSR*, *Mekh. Zhidk. Gaza*, No. 2, 18–26 (1994).
16. I. G. Lebo, V. V. Nikishin, V. B. Rozanov, and V. F. Tishkin, On the effect of boundary conditions on the instability growth at a contact surface in passage of a shock wave, *Bull. Lebedev Phys. Inst.*, No. 1, 40–47 (1997).
17. W. K. Anderson, J. L. Thomas, and B. Van Leer, Comparison of finite volume flux vector splitting for the Euler equations, *J. AIAA*, **24**, No. 9, 1453–1460 (1986).
18. G. A. Ruev, A. V. Fedorov, and V. M. Fomin, Development of the Richtmyer–Meshkov instability in interaction of a diffusion layer of mixing of two gases with transmitted and reflected shock waves, *Prikl. Mekh. Tekh. Fiz.*, **51**, No. 3 (2010) (in press).
19. M. Brouillette and B. Sturtevant, Growth induced by multiple shock waves normally incident on plane gaseous interfaces, *Phys. D*, **37**, Nos. 1–3, 248–263 (1989).
20. M. Brouillette and B. Sturtevant, Experiments on the Richtmyer–Meshkov instability: single-scale perturbations on a continuous interface, *J. Fluid Mech.*, **263**, 271–292 (1994).